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Annuities on Lives. By CHARLES M. WILlich, Esq., Actuary
to the University Life Assurance Society.

[Read before Section F (Economic Science and Statistics), at the Meeting of
the British Association for the Advancement of Science, 1857.]

THE tables of the values of annuities on lives, as hitherto constructed, only show the rate of interest which a purchaser may make on the money employed, and replace the *capital*, provided he can reinvest the surplus income, beyond the interest on the *purchase money*, at the same rate of interest.

As it is often important to know the price which should be paid for such annuities, in order that the purchaser may enjoy a certain higher rate of interest on the money invested, whilst the circumstances of the times only permit a much lower rate to be calculated upon for the future investments, I have constructed the following formula, which will show the purchase money to be paid for an annuity under such circumstances.

Let \mathfrak{A} = annuity on the given life.

x = age.

r = interest on £1 for one year, as used in the Table of Annuities, and at which rate the reinvestments are to be made.

r' = interest of £1 for one year required by the purchaser, for the use of his money.

Formula.— $\left(\frac{1}{\mathfrak{A}_x - r + r'}\right)$ = present worth of annuity on a life aged x ,

so as to secure a certain higher rate of interest on the capital employed, while the surplus income is invested at the rate at which the annuity table is constructed.

The same principle applies at various rates of interest, and to annuities for terms of years certain, or annuities for joint lives, or the longest of two lives, &c.

I beg to annex a table, with an example, showing the mode of construction.

TABLE OF ANNUITIES, according to the Carlisle Table of Mortality, at 3 per cent.; also the price which should be paid for an Annuity, so that the purchaser may obtain 5 per cent. Interest on the money invested, while the reinvestments to replace capital are estimated to be made at only 3 per cent.

Age.	Value of Annuity at 3 per Cent. Carlisle.	Value of Annuity if 5 per Cent. is to be enjoyed, while the Reinvestments are only made at 3 per Cent.
20	21·695	15·130
30	19·557	14·058
40	17·142	12·765
50	14·302	11·121
60	10·491	8·671
70	7·123	6·235
80	4·365	4·014
90	2·499	2·380

Example.—At the age of 20, the value of an annuity, Carlisle 3 per Cent. Table, is 21·69528 years' purchase; the log. of which is 1·3363652, and the complement is ·6636348, and the equivalent number is 4·6093, which represents the percentage that every £100 would receive annually. This sum of 4·6093 is composed of £3 for the interest on £100, and £1·6093, which being reinvested annually at 3 per cent. during the life of 20, will replace the £100.

As we wish to receive £5 per cent. instead of £3, we must add 2· to the 4·6093, making 6·6093, which is the percentage we ought to get in order that we may enjoy 5 per cent. while the reinvestments are made at 3 per cent. We find that the log. of 6·6093 is ·8201555, and the complement is ·1798445—the equivalent number being 15·130, or the value of an annuity at the age of 20, which will afford 5 per cent. to be enjoyed; while the surplus (1·6093), accumulated at 3 per cent., will replace the capital, if the party lives according to the experience of the Carlisle Table of Mortality.

[NOTE.—In the Note at page 102, vol. i., of this *Journal*, we called attention to the advantages to be derived from the construction of a table showing the annual sum required, at practicable rates, to produce £1 at the end of any number of years. The remarks there made are applicable in the present instance: for, let r' be the annual sum required to produce £1 at death, and r the rate to be realized during life, then, as

$r' + r : 1 :: 1 : \frac{1}{r' + r}$, an expression equivalent to the one above given.—

ED. A. M.]